Dialogue Social Science Review (DSSR)
www.thedssr.com

ISSN Online: 3007-3154
ISSN Print: 3007-3146

DIALOGUE SOCIAL SCIENCE REVIEW

Vol. 3 No. 5 (May) (2025)

Modeling Relationships of Affective Factors on Grade Eight Students' Mathematics Achievement

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Abstract
The current study employed a quantitative research design to explore the intricate network of affective factors linked with academic achievement in mathematics among 8th-grade students in Karachi. A total of 160 students (80 boys and 80 girls) were selected from ten randomly selected private schools. The study reviewed the contemporary literature highlighting the paramount role of affective traits for academic success in mathematics and presented the multi-dimensional relations among motivation, attitude, belief, academic engagement, and academic achievement. With the help of structural equation modeling, the study assessed the direct and indirect relationships of factors affecting achievement, thereby revealing the underlying complexity within the framework of a theoretical model. The results show a significant relationship of students' self-beliefs with their motivations, engagement, and achievement in mathematics. The study also explored the moderating role of belief about the teacher in the relationship between affective variables and mathematics achievement, thereby emphasizing the importance of the teacher-student positive relationship in enhancing mathematics achievement. The findings have implications for the creation of supportive learning environments that foster mathematics achievement as well as for the development of evidence-based interventions that may enhance mathematics education in urban schools in Karachi.

Keywords: Beliefs, motivations, engagement, mathematics achievement

Introduction
The middle school years, up until grade eight, are crucial for children because they have a big impact on what they will study at the secondary level and what careers they will pursue in the future. Being a core school subject, mathematics directly affects several cognitive student's skills. The importance of mathematics is also due to its crucial role in students' academic and future job aspirations. It is fundamental to building problem-solving and analytical skills as well as the basis for many fields. Nevertheless, despite how important it is, many children struggle to succeed in maths. In the past, the role of affective (emotional) components has been undervalued in favour of practicing pupils' cognitive math skills. Recent research, however, has shown that affective elements have a major impact on

children's maths ability, even though cognitive abilities and home environments are acknowledged as important determinants of achievement. To explore the effects of motivation, attitude, belief, and academic engagement on eighth-grade students' mathematics achievement at private schools in Karachi, the purpose of this research proposal is to develop a comprehensive model.

The major affective traits that impact mathematics performance are motivation, attitude, belief, and academic engagement, in addition to their cognitive abilities. Although teachers generally concentrate on improving their students' cognitive math skills, it is equally important to recognize and address the importance of affective elements. The goal of this study is to model how these affective factors relate to students' mathematical proficiency to get important knowledge for educational interventions.

General Objective

The general objective of this study is to develop a Structural Equation Model (SEM) model to test the relationship of affective traits of students with their mathematical achievement at the grade eight level in Karachi.

Specific Objectives

The study has the following specific objectives:

1. To examine the relationships between motivation, attitude, belief, academic engagement, and mathematics achievement among grade eight students.
2. To develop a comprehensive model using Structural Equation Modeling (SEM) to test and estimate the hypothesized relationships.
3. To identify the relative importance of each affective factor in predicting mathematics achievement.

Literature Review

Mathematics is a core subject in the school curriculum, and has a great significance in the development of diverse cognitive skills. It serves as a foundation for numerous fields, making it crucial for future success in both academic and professional endeavours (OECD, 2019). The underachievers in mathematics have a higher probability of dropping out of school or avoiding careers in the field (Rozel et al., 2015). Nitzan-Tamar and Kohen (2022) assert that mathematics is a major factor in determining career prospects, especially in the STEM fields. Therefore, researchers and educators need to pay attention to those factors that influence students' math performance. In this way, students can receive the right instruction depending on their individual needs.

Math achievement is particularly important during the elementary school years, which run from Grades 6 through 8. At this level, success in this subject influences curricular choices and admission to advanced science and mathematics programs (Singh et al., 2002). Therefore, early interest and success in mathematics are essential for elementary school students. Despite its critical importance, there is little research available in Pakistan on the school and individual factors that influence adolescent academic achievement in this subject. Therefore, examining the factors that influence academic success in mathematics, such as school motivation, academic engagement, and beliefs about self, teacher, and mathematics is very important.

In the last three decades, a substantial body of research has examined and correlated students' emotional traits with their success in mathematics all over the world (Hannula, 2016). Fong and Kremer, (2020) reported that mathematics achievement and persistence are positively correlated with motivation. Ma and Kishor (1997) assert that students' attitudes toward mathematics have a significant influence on their willingness to work hard to learn the subject and, ultimately, their achievement in it.

According to Bandura (1997), students' academic achievement may be significantly impacted by their perceptions of their abilities and the importance of mathematics in their daily lives. Students who possess higher levels of expectancy for success and/or perceived value for the task are more likely to participate in the learning process, even when faced with challenges (Murayama et al., 2013).

Students' beliefs about their teacher's emotional support are positively correlated with their math performance (Yang et al., 2021). Better math performance has also been linked to academic engagement, which measures students' feelings of ownership and active participation in the learning process (Jang et al., 2010). Teachers and policymakers, therefore, need to understand the significance of these affective components to design effective interventions that raise children's achievement in mathematics.

To investigate the connections between affective aspects and mathematical ability, several theoretical models have been put forth. The Expectancy-Value Theory (EVT) developed by Eccles and Wigfield (2002) states that students' expectations of success and the value they place on the subject have an impact on their motivation and achievement-related decisions. In the context of the EVT framework, increasing students' self-efficacy and highlighting the importance of mathematics are essential strategies for raising math achievement. Self-Determination Theory (SDT) by Deci and Ryan (2000), emphasizes how competence, autonomy, and intrinsic motivation drive students' academic engagement. Giving students the chance to learn autonomously and with a sense of purpose in the context of mathematics can boost their motivation and performance (Garon-Carrier et al., 2016).

Keeping in view the sizeable research gap in studying affective factors about mathematics achievement, especially at the elementary level in Pakistan, this study has been conducted. Partial Least Square Structural Equation Modeling (PLS-SEM) has been employed using Smart PLS-4 to study the complex relationship among different affective factors that lead to mathematics achievement. Partial Least Square (SEM) enables a better model formulation and estimate which includes formative constructs (Becker et al., 2012).

By creating a solid model that includes beliefs, motivations, and academic engagement to thoroughly explore their influence on grade eight students' mathematical achievement. The study has made use of PLS-SEM to provide a more in-depth investigation of the connections between these affective characteristics and mathematics achievement, taking into account their contributions and identifying the relative importance of each affective factor in predicting mathematics achievement. The theoretical links between the many affective variables that are believed to impact mathematical achievement will be tested and estimated by the suggested model. This study is significant because it has the potential to provide educators and policymakers with useful data to aid in the development of targeted interventions that address affective components and improve math performance in grade eight students attending private schools in Karachi.

Methodology

The study employed a cross-sectional design and a quantitative methodology. Data were gathered from a sample of 160 grade eight students from various private schools in Karachi, 80 of whom were boys and 80 of whom were girls. At first, fifteen schools were chosen at random from three Karachi districts. Districts Central, East, and South of Karachi each have five schools. Ten schools expressed interest and granted consent when asked, out of all the schools. A purposive sample design was used to choose two math teachers for grades eight from each of these ten schools. Four students—two boys and two girls—were asked to be chosen by each teacher from their classes. One high achiever and one low achiever from the boy gender, as well as one high achiever and one low achiever from the girl gender, were the selection criteria for the students. Thus, eight boys and eight girls were chosen in this manner from each school.

Instrument

Two sections of a questionnaire were used to collect data from these students. The first section asked about four constructs, which were Belief About Mathematics (BAM), Belief About Self (BAS), Belief About Teacher (BAT), and Math Value (MV). The second section, which was intended for their teachers, included the following: Mathematics Engagement (MEng), Mathematics Achievement (MAch), Motivation-1 (general motivation towards studies), and Motivation-2 (motivation towards mathematics) (grades earned by students in mathematics in class VI and class VII final exams). The student completed the first part of the questionnaire, and in each case, the teacher completed the second section. Data were collected by making in-person visits to every school. Partial Least Squares-based Structural Equation Modeling (PLS-SEM) in SMART-PLS (version 4) was used to analyze the collected data.

Measurement Model

Confirmatory Factor Analysis (CFA) was performed using Smart PLS (version 4) to test the measurement models. As part of confirmatory factor analysis, factor loadings were assessed for each item, two items from Belief About Mathematics (BAM1 & BAM3), one item from Belied About Self (BAS3), four items from Belief About Teacher (BAT3, BAT4, BAT5, & BAT6), and five items from Math Value (MV1, MV2, MV6, MV7, & MV8) were removed due to low factor loadings ($< .50$). Different model fit indices calculated by Smart PLS were used to assess the overall goodness of fit of proposed model (CMIN/df, GFI, CFI, TLI, SRMR, and RMSEA). All values for the goodness of fit of a model were within the acceptable range (Ullman, 2001; Hu & Bentler, 1998; Bentler, 1990). The model comprising eight constructs (BAS, BAT, BAM, MV, M1, M2, MEng, and MAch) was found good fit (Table 1) for the data CMIN/df = 2.728, GFI = .965, CFI = 0.904, TLI = 0.912, SRMR = 0.071, and RMSEA = 0.068.

Nunnally and Bernstein (1994) proposed that the value of Cronbach's Alpha should be greater than 0.70 for each construct. According to Hair et al. (2010), the value of composite reliability should be greater than 0.70 for each construct. The values of Cronbach's Alpha and composite reliability were within the

ISSN Online: 3007-3154
ISSN Print: 3007-3146

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acceptable range. Therefore, construct reliability was established for each construct in the study (Table 2).
Fornell and Larcker (1981) reported that to assess the convergent validity of a construct, the value of Average Variance Extracted (AVE) should be above 0.50. The convergent validity of the constructs used in this study was achieved (Table 2) because all the values of AVE were above the proposed threshold (AVE > 0.50).
For discriminant validity, Fornell and Larcker's criterion is that the square root of AVE for a construct should be greater than its correlation with the other constructs in the study. Another method to assess the discriminant validity is the HTMT ratio which is increasingly used these days. In the HTMT ratio method, all ratios should be less than the threshold of .85 (Henseler et al., 2015).
The discriminant validity of all the constructs was assessed using both the Fornell and Larcker criterion and the Hetrotrait-Monotrait (HTMT) ratio. Both Fornell and Larcker criterion and HTMT ratio threshold were met and thus discriminant validity was established. The results of discriminant validity are presented in Table 3 (Fornell and Larcker Criterion) and Table 4 (HTMT Ratio).

Table 1: *Fit Indices*

Fit Indices	Recommended Value	Source	Obtained Value
p	insignificant	Bagozzi and Yi (1988)	0.063
CMIN/df	3 - 5	Less than 2 (Ullman, 2001) up to 5 (Schumacker & Lomax, 2004)	2.728
GFI	> .90	Hair et al (2010)	0.965
CFI	> .90	Bentler (1990)	0.904
TLI	> .90	Bentler (1990)	0.912
SRMR	< .08	Hu and Bentler (1998)	0.071
RMSEA	< .08	Hu and Bentler (1998)	0.068

Table 2:*Loadings, Reliability, and Convergent Validity*

Indicators	Loadings	Cronbach's Alpha	Composite Reliability	Average Variance Extracted (AVE)
BAM2	0.868	0.763	0.764	0.592
BAM4	0.798			
BAS1	0.856	0.768	0.763	0.621
BAS2	0.839			
BAS4	0.703			
BAT1	0.805	0.862	0.866	0.611
BAT2	0.707			
BAT7	0.620			
BAT8	0.720			
BAT9	0.856			
BAT10	0.885			
M11	0.709	0.819	0.845	0.572
M12	0.802			
M13	0.929			

ISSN Online: 3007-3154
ISSN Print: 3007-3146

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M14	0.770			
M21	0.731	0.848	0.850	0.738
M22	0.804			
M23	0.848			
M24	0.882			
MEng1	0.867	0.968	0.968	0.812
MEng2	0.947			
MEng3	0.897			
MEng4	0.931			
MEng5	0.916			
MEng6	0.929			
MEng7	0.921			
MV3	0.846	0.794	0.805	0.584
MV4	0.791			
MV5	0.736			
MV9	0.747			

Table 3: *Fornell & Larcker Criterion*

	BAM	BAS	BAT	M1	M2	MEng	MV
BAM	**0.627**						
BAS	0.609	**0.702**					
BAT	0.621	0.692	**0.722**				
M1	0.622	0.687	0.696	**0.756**			
M2	0.600	0.651	0.669	0.719	**0.724**		
MEng	0.598	0.633	0.714	0.696	0.689	**0.901**	
MV	0.601	0.692	0.695	0.631	0.688	0.604	**0.696**

Note. Bold values are square root of AVE

Table 4: *HTMT Ratio*

	BAM	BAS	BAT	M1	M2	MEng	MV
BAM	0.701						
BAS	0.731	0.884					
BAT	0.323	0.329	0.325				
M1	0.479	0.559	0.450	0.765			
M2	0.462	0.512	0.349	0.699	0.783		
MEng	0.849	0.417	0.399	0.582	0.501	0.510	
MV							

Data Analysis & Results

Figures 1. show the structural model. Table 5-8 provides the results of the moderation and mediation analyses.

ISSN Online: 3007-3154
ISSN Print: 3007-3146

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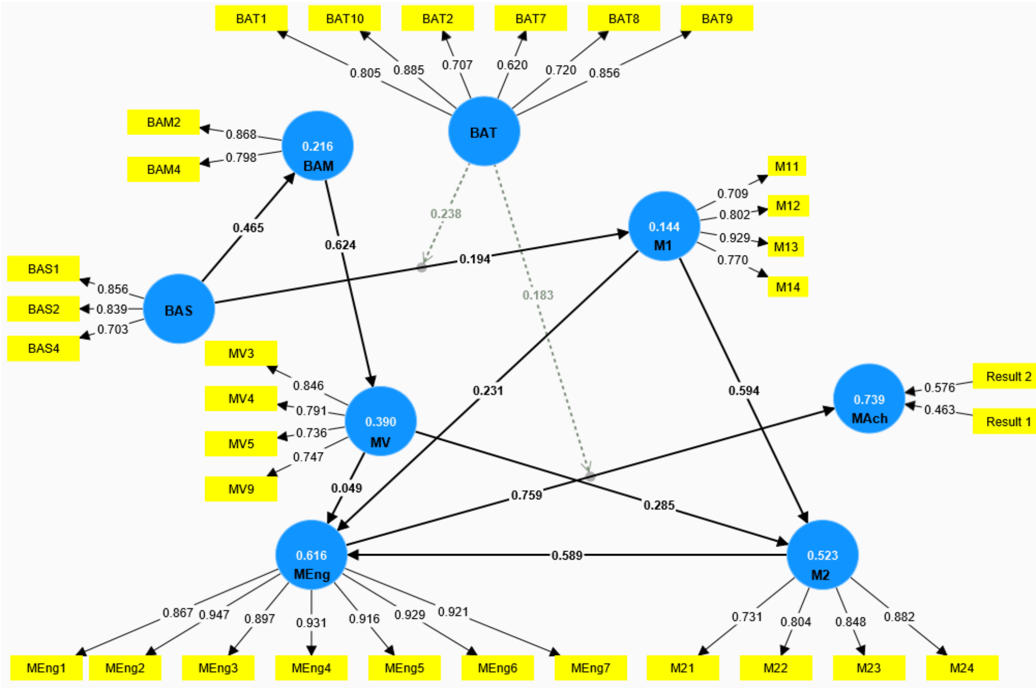

Figure 1. Structural Model

Path Analysis

Table 5: *Direct Effects*

Path	β	t-value	p-value	Result
BAM → MV	0.624	9.975	0.000	Supported***
BAS → BAM	0.465	5.756	0.000	Supported***
BAS → M1	0.194	1.253	0.179	Not Supported
M1 → M2	0.594	7.105	0.000	Supported***
M1 → MEng	0.231	1.695	0.045	Supported*
M2 → MEng	0.589	5.115	0.000	Supported***
MEng → MAch	0.759	12.050	0.000	Supported***
MV → M2	0.285	3.704	0.000	Supported***
MV → MEng	0.049	0.599	0.275	Not Supported

*Note. BAS = Belief about Self, BAM = Belief about Mathematics, MEng = Mathematics Engagement, MAch = Mathematics Achievement, MV = Mathematics Value, M1 = Academic Motivation, M2 = Motivation towards Mathematics, *p < .05, **p < .01, ***p < .001*

Table 5 shows the direct effects within the structural model. Seven of the nine paths were significant. The direct effect of Belief About Self (BAS) on Motivation Towards Studies (M1) was not significant (β = 0.194, t = 1.253, p = .179). Similarly, the direct effect of Math Value (MV) on Math Engagement (MEng) was not significant (β = 0.049, t = 0.599, p = .257).
The significant direct effects are revealing meaningful relationships among the variables in the model. However, the non-significant direct effects of BAS on M1 and MV on MEng suggest more complex relationships or potential mediation.

Further exploration of indirect effects is necessary to better understand the dynamics of these associations.

Table 6: *Indirect Effects*

Path	β	t-value	p-value	Result
BAS → M1 → M2 → MEng → MAch	0.051	1.134	0.128	Not Supported
BAM → MV → M2 → MEng → MAch	0.079	3.014	0.001	Supported**
BAS → BAM → MV→ M2→ MEng→ MAch	0.037	2.376	0.009	Supported**
BAS → BAM → MV → MEng → MAch	0.011	0.545	0.293	Not Supported
BAS → BAM → MV → M2 → MEng	0.049	2.414	0.008	Supported**

*Note. BAS = Belief about Self, BAM = Belief about Mathematics, MEng = Mathematics Engagement, MAch = Mathematics Achievement, MV = Mathematics Value, M1 = Academic Motivation, M2 = Motivation towards Mathematics, *p < .05, **p < .01*

Table 6 presents the indirect effects of the structural model. First, the indirect effect from Belief About Math (BAM) through Math Value (MV) to Motivation Towards Math (M2) and Math Engagement (MEng) through Math Achievement (MAch) was statistically significant (β = 0.079, t = 3.014, p < .001). This particular pathway highlights the essential nature of positive beliefs held about math and its subsequent influence on motivation, engagement, and ultimately, achievement in mathematics. Furthermore, the indirect effect from Belief About Self (BAS) to Belief About Math (BAM) through Math Value (MV) to Motivation Towards Math (M2) and Math Engagement (MEng) through Math Achievement (MAch) was statistically significant (β = .037, t = 2.376, p < .05). This pathway is illustrative of the complex dynamics of self-beliefs, math values, motivation, engagement, and ultimately, math achievement.

Moderation Analysis

H1: Belief About Teacher (BAT) positively moderates the positive relationship between Belief About Self (BAS) and Motivation towards studies (M1) such that increased belief about teacher increases the relationship between BAS and M1.

H2: Belief About the Teacher (BAT) positively moderates the positive relationship between Math Engagement (MEng) and Math Achievement (MAch) such that increased belief in the teacher increases the relationship between MEng and MAch.

The study assessed the moderating role of Belief About Teacher (BAT) on the relationship between Belief About Self (BAS) and Motivation toward Studies (M1). Without the inclusion of the moderating effect (BAT*BAS), the R-squared value of M1 was 0.092. This shows that a 9% change in M1 is accounted for by BAS. With the inclusion of the interaction term, R-Square increased to 0.180. This shows an increase of 9% in variance explained in the dependent variable (M1) is due to the moderator.

ISSN Online: 3007-3154
ISSN Print: 3007-3146

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Further significance of the moderating effect was analyzed, and the results revealed a positive moderating effect of BAT on the relationship between BAS and M1 ($\beta = 0.264$, $t = 2.691$, $p < .05$) supporting H1. This shows that with an increase in Belief About Teacher (BAT), the relationship between BAS and M1 is strengthened. The moderation analysis summary is presented in Table 7.
The f-square effect size was 0.017, according to Kenny's (2018) proposition, 0.005, 0.01, and 0.025 constitute small, medium, and large effect sizes of moderation respectively. This is a large moderation effect. Hence it is confirmed that there is a positive moderation effect experienced in the model stating BAT strengthens the relationship between BAS and M1.
The study also assessed the moderating role of Belief About Teacher (BAT) on the relationship between Math Engagement (MEng) and Math Achievement (MAch). Without the inclusion of the moderating effect (BAT*MEng), the R-Square value of MAch was 0.702. This shows that 70% of the change in MAch is accounted for by MEng. With the inclusion of the interaction term, R-Square increased to 0.739. This shows an increase of 3.9% in variance explained in the dependent variable (MAch) is due to the moderator.
The results revealed a positive moderating effect of BAT on the relationship between MEng and MAch ($\beta = 0.183$, $t = 2.990$, $p < .05$) supporting H2. F-square effect size was 0.092, which is a large moderation effect. This shows that with an increase in Belief in Teachers (BAT), the relationship between MEng and MAch is strengthened. The moderation analysis summary is presented in Table 7.

Table 7: *Moderation Analysis*

Relationship	β	SE	t value	p-value
Moderating Effects				
(BAT*BAS) → M1	0.264	0.098	2.691	0.004*
(BAT*MEng) → MAch	0.183	0.061	2.990	0.001*

Note. SE: Standard Error, BAS = Belief about Self, BAT = Belief about Teacher,
*MEng = Mathematics Engagement, MAch = Mathematics Achievement, *p < .05, **p < .01*

Moderated Mediation

The study assessed the moderated mediation effect of Belief about Teacher (BAT) on the relationship between Belief about Self (BAS) and Motivation towards Studies (M1). M1 further acted as a mediator between BAS and motivation towards math (M2). Without the inclusion of the moderating effect (BAT*BAS), the R-Square value of M1 was 0.065, and that of M2 was 0.514. With the inclusion of the interaction term, R-Square increased to 0.144 (M1) and 0.523 (M2). This shows an increase of 8% in variance explained in M1 and 1% in M2 is due to the moderator. The results (Table 8) reveal that the moderated mediation effect ($\beta = 0.141$, $t = 2.352$, $p < .05$) effect is significant. This shows that with an increase in Belief in Teacher (BAT), the relationship between BAS and M1 is strengthened which further positively affects M2. This implies that BAT positively affects the positive relationship between BAS and M1 which further affect positively to M2.
The moderated mediation path (BAT × BAS → M1 → M2) was further tested till MAch and was found significant ($\beta = 0.063$, $t = 1.982$, $p < .05$). It means that M2

further positively affect MEng as a mediator, and MEng positively affect MAch.

When the moderated mediation path (BAT × BAS → M1) was further assessed till MAch without taking M2 as a mediator, it was found insignificant (β= 0.042, t= 1.314, p = .094). It means that without M2, the positive effect of M1 on MEng as a mediator, and further MEng's effect on MAch is not significant. It implies that M2 (motivation towards math) is a significant mediator between M1 (general motivation towards studies) and MEng (math engagement). The moderated mediation analysis summary is presented in Table 8.

Table 8: *Moderated Mediation Analysis*

Relationship	β	SE	t value	p-value
Moderated Mediation Effects				
BAT × BAS → M1 → M2	0.141	0.060	2.352	0.009**
BAT × BAS → M1 → M2 → MEng → MAch	0.063	0.032	1.982	0.024*
BAT × BAS → M1 → MEng → MAch	0.042	0.033	1.314	0.094

Note. SE: Standard Error, BAS = Belief about self, BAT = Belief about teacher,
*MEng = Mathematics Engagement, MAch = Mathematics Achievement, *p < .05, **p < .01*

Discussion

The purpose of this study was to develop a Structural Equation Model (SEM) model to test the relationship of affective traits of students with their mathematical achievement at the grade eight level in Karachi. A substantial body of research in mathematics education has examined students' affective traits about their achievement level (Hannula, 2016; Jang et al., 2010; Yang et al., 2021). The present study, in the context of recent research, proposed a comprehensive model to estimate the hypothesized relationships among students' Beliefs (Belief About Self, Belief About Teacher, Belief About Math, Math Value), Motivations (Motivation towards Studies, Motivation towards Math), Math Engagement, and Mathematics Achievement. The results of this study, consistent with previous studies (Hannula & Maijala, 2017), emphasize the critical role of affective traits of students for achievement in mathematics (Hannula, 2016) at the grade eight level.

M2 (Motivation towards Math) emerged as a key determinant of mathematics achievement in this model, which is consistent with the assertion of Ma and Kishor (1997) that students' motivation towards studying mathematics profoundly affects their success in the subject. The results of this study are also consistent with the Expectancy-Value Theory (EVT) (Eccles & Wigfield, 2002), which highlights that students' expectations of success and their belief in the value of mathematics influence their motivation toward this subject. Belief About the Teacher (BAT) emerged as another important factor that moderates the relationship between Math Engagement (MEng) → Math Achievement (MAch) as well as between Belief About Self (BAS) → Motivation towards Studies (M1). Belief In a Teacher (BAT) is the expectation of success under the guidance of their teacher and the belief of students in their teacher that he/she is competent enough to support them in the process of their learning. These results also support the findings of Yang et al. (2021) by highlighting the necessity of taking into consideration students' beliefs about their teacher for motivation, engagement, and achievement. The findings of Murayama et al. (2013) are also coherent with these results.

Finally, MEng is also a mediator between M2 and MAch, which stresses the importance of enhancing students' engagement in mathematics by increasing their intrinsic motivation towards mathematics. Combining these results with the principles of self-directed learning is easy: Math Engagement can be interpreted as a tool for providing students with opportunities for self-directed learning. One of the opportunities may be autonomy in choosing whether to complete the self-selected tasks, which has proved to be successful in enhancing students' motivation and engagement. The results are also consistent with Deci and Ryan's self-determination theory (2000), which accentuates the importance of competence, autonomy, and intrinsic motivation in fostering students' academic engagement. Similarly, the results are in line with the findings of Jang et al. (2010) and Garon-Carrier et al. (2016).

The use of structural equation modeling (SEM), in particular partial least squares SEM (PLS-SEM) in this study, has helped explore the complex relationships among beliefs, motivations, academic engagement, and academic achievement in mathematics. The path analysis has shown significant direct and indirect effects in the model. The most significant indirect effect (BAS → BAM → MV→ M2→ MEng→ MAch) shows a clear progression of affective traits towards mathematics achievement. Self-belief is positively correlated with belief in mathematics, which further positively correlates with mathematics value. Math value in turn positively affects motivation towards mathematics which positively affects engagement which finally affects math achievement.

Findings

The findings of this study reveal some complex relationships between affective factors and mathematics achievement of grade eight students attending private schools in Karachi. Drawing from the recent studies highlighting the key role of affective factors in mathematics achievement (Hannula, 2016), this study was focused on students' beliefs (self-belief, belief on teacher, belief about subject, belief about value of math), motivations (motivation towards studies in general & motivation towards mathematics), their engagement with mathematics and their performance as significant elements of students' learning experiences to identify the relative importance of each affective factor and relationships among these in predicting mathematics achievement (Hannula & Maijala, 2017).

Although direct effects between BAS → M1, as well as MV → MEng, were non-significant, this study revealed significant direct effects (BAM → MV; BAS → BAM; M1 → M2; M1 → MEng; M2 → MEng; MEng → MAch; MV → M2). The study revealed significant indirect effect, particularly, the indirect effect of (BAS → BAM → MV→ M2→ MEng→ Mach), which emphasized the importance of positive self-beliefs and their positive impact on belief about the nature of math, which in turn influences positive beliefs about math and motivate students to pursue the subject, which in turn affects engagement and, ultimately, mathematics achievement. The insignificant, indirect effect of (BAS → BAM → MV → MEng → Mach) reveals the significance of M2 (Motivation towards Math) between MV (Math Value) and MEng (Math Engagement) as in the presence of M2, the path is significant. The path (BAM → MV → M2 → MEng → Mach) is significant but the path (BAS → M1 → M2 → MEng → Mach) was not significant, which implies that BAS (Belief About Self) in the absence of BAM (Belief About Math) and MV (Math Value) does not significantly affect math achievement.

In addition to indirect effects, this study explored the moderating effect of Belief About Teacher (BAT) on the relationships between BAS and M1, as well as between Math Engagement (MEng) and Math Achievement (Mach). The positive moderating effects of BAT highlighted the significant influence of students' beliefs about their teacher in strengthening the relationship between their self-beliefs and motivation toward studies. This implies that a positive and supporting outlook of teachers enhances the students' motivation towards studies. Moreover, the positive moderating effects of BAT on Math Engagement (MEng) and Math Achievement (Mach) highlight the importance of supportive teacher-student dynamics in enhancing mathematics achievement.

The complete moderated mediation path (BAT × BAS → M1 → M2 → MEng → MAch) was significant which shows that Belief About Teacher (BAT) positively affects the whole path but without M2 the path was not significant. This signifies the role of M2 (motivation towards math) as a substantial mediator between M1 (general motivation towards studies) and MEng (math engagement).

Conclusion

In conclusion, this study makes valuable contributions to the growing literature, which underscores the immense importance of affective factors in mathematics achievement. By revealing how motivation, attitude, belief, and academic engagement are interwoven and intertwined, the findings of this study offer valuable implications for educators and scholars hoping to develop strategies to promote a positive learning environment that is attuned to supporting grade eight students' mathematics achievement.

Based on the findings of this study it is recommended that mathematics teachers should be provided professional development opportunities with a focus on fostering positive attitudes and beliefs among students. The curriculum of mathematics should be designed and executed with an approach that promotes positive attitudes towards mathematics and active student engagement in the learning process. It is also recommended to explore individual student beliefs, motivations, and engagement levels in mathematics to create a supportive learning environment conducive to academic success. Further research, with a large sample size, and in diverse educational settings is recommended to further explore the complex relationship of affective factors with mathematics achievement.

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